PERGAMON

International Journal of Solids and Structures 36 (1999) 3073–3084

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

Direct fractal measurement of fracture surfaces

Heping Xie,* Jin-an Wang

Institute of Rock Mechanics & Fractals, Beijing Graduate School, China University of Mining and Technology, Beijing 100083, P.R. China

Received 24 October 1997; in revised form 14 April 1998

Abstract

To overcome the difficulties in estimation of fractal dimension for fracture surfaces, a new method of fractal measurement—the projective covering method (PCM) is proposed in this paper. Based on the measurements using a laser scanner, the fractal dimension $D_s \in [2, 3)$ of fracture surface is directly estimated. The research results agree with the theory of fractal geometry and measurement data.

Keywords: Projective covering method; Fracture surface; Fractal dimension

1. Introduction

For many years, fracture surfaces have been described by statistical parameters following the metrology used in tribology and contact mechanics (Johnson, 1985). The parameters can be classified into three categories according to the type of characteristic that they measure:

(1) Amplitude parameters, such as centerline average value, mean square value, root mean square (RMS) value, mean square of the first derivative, RMS of the first derivative (Z_2), RMS of the second derivative (Z_3), percentage excess of distance (Z_4).
(2) Spacing parameters, such as autocorrelation function, spectral density function, structure function (SF), roughness profile index (R_p)
(3) Hybrid parameters, micro-average *i* angle etc.

These parameters are not only quite complicated, but also suffer from scale effect, i.e., the estimated values of roughness depend on the length of sample, the digitizing intervals and the resolution of instrument. Based on extensive experiments, Barton and Choubey (1977) proposed

* Corresponding author. Fax: 0086 10 62325016

PII: S0020–7683(98)00141–3

a conceptual model to quantify the surface roughness of rock joints. Accordingly, the roughness is classified into ten groups, and the joint roughness coefficients (JRC) ranges from 0–20. This model has been recommended for years by ISRM and adopted for a good period of time in the practice of rock engineering.

Since Mandelbrot (1967, 1983) introduced fractal geometry, many investigations have tried to interpret JRC by fractal dimension (Lee et al., 1990; Mearz and Franklin, 1990; Turk et al., 1990; Muralha, 1992; Wakabayashi and Fukushige, 1992; Xie, 1993, 1996; Xie and Pariseau, 1994). Fractal theory describes an object with irregular shape, or a physical quantity or natural phenomenon with irregular distribution in a quantitative manner. The property of fractal geometry can be mathematically expressed by the concept of self-similarity and self-affinity, which suggests that when the shape of an object is magnified more and finer structure can be recognized. Fractal dimension is scale-invariant providing geometric structure at all scales.

In fact, natural fracture surfaces rarely show self-similar fractal property (Xie et al., 1996, 1997a). Different definitions of fractal dimension, fractal measurement techniques and scale parameters may produce different values of fractal dimension even for the same fracture surface. Perhaps, it is the reason that many controversial findings have been reported in recent literatures of fractal characterization of Barton's standard JRC profiles (Miller et al., 1990; Odling, 1994; Outer et al., 1995). A relation between roughness and the fractal dimensions is not straightforward and cannot be estimated without conditions for sampling parameters, resolution of instrument and measurement methods. Any conclusion on the fact that fractals do or do not exist in rough surfaces should be taken with care and the fractal characterization of fracture surface as fractal regime is at least very doubtful (Outer et al., 1995).

The most critical problem, however, is that a real fracture usually extends in a spatial plane. In general, it is very difficult to make a direct measurement for a rough surface. Most of the fractal characterization of a rough surface, however, had to employ indirect methods, such as slit island (SI), spectrum, and variogram to measure a sectional profile. Fractal dimension measured by these methods ranges $D \in [1, 2)$. Mandelbrot (1983) suggested that fractal dimension of a topographic surface can be obtained by adding 1.0 to the fractal dimension from a single profile of that surface. Investigation (Wang et al., 1996; Xie et al., 1997b) shows, however, that fractal dimensions vary from one sectional profile to another and also they differ in different directions over the fracture surface. Since the anisotropy and heterogeneity of fracture surface structure, fractal measurement based on profiles is questionable as follows: (1) whether a rough surface could be simulated by a profile; (2) which sectional profile and along which direction of the surface could be warrentedly consulted.

To find out a solution, a new fractal measurement method—Projective Covering Method (PCM)— is proposed for direct estimation of real fractal dimension $D_s \in [2, 3)$ for a fracture surface. The primary results promote the validity of PCM as applied for description of roughness of rock fracture surfaces.

2. The projective covering method

As is well known, covering method is one of the most common methods for fractal measurement. It is suitable not only for simple fractals but also for complex fractals (Falconer, 1990: Feder, 1988). However, it appears impossible using such a method to cover a fractal surface in a direct manner. The real fractal dimension $D_s \in [2. 3)$ have been replaced by using approximate fractal dimensions $1 < D < 2$ which is obtained from the sectional profile measurement. In the present work, we propose a new method of fractal estimation—the projective covering method (Fig. 1). From this method, the real fractal dimension $D_s \in [2, 3)$ for a fracture surface can be directly measured.

Symbols A and B in Fig. 1a denote, respectively, a real fracture surface and the corresponding projective network covering the surface. When kth square *abcd* (a, b, c and d are the four points of the square) with a selected scale of $\delta \times \delta$, the heights of a fracture surface at points a, b, c and d correspond to h_{ak}, h_{bk}, h_{ck} and h_{dk} (Fig. 1b). Accordingly, the area of rough surface surrounded by points *abcd* can be approximately calculated by

$$A_k(\delta) = \frac{1}{2}\{[\delta^2 + (h_{ak} - h_{dk})^2]^{1/2}[\delta^2 + (h_{dk} - h_{ck})^2]^{1/2} + [\delta^2 + (h_{ak} - h_{\mathrm{bk}})^2]^{1/2}[\delta^2 + (h_{bk} - h_{ck})^2]^{1/2}\}. \tag{1}$$

The entire area of the rough surface under kth scale measurement is given by

$$A_T(\delta) = \sum_{K=1}^{N(\delta)} A_k(\delta) \tag{2}$$

where, $N(\delta)$ is the total number of cells with scale of $\delta \times \delta$ needed to cover the rough surface. Obviously, the measured area $A_T(\delta)$ of a rough surface depends on δ. A smaller δ yields a greater $A_T(\delta)$. As $\delta \to 0$, $A_T(\delta)$ approximates to a real area of the rough surface.

In fractal geometry, the measure of a fractal object in E-dimensional space can be expressed in a general form (Xie, 1993)

$$G(\delta) = G_0 \delta^{E-D} \tag{3}$$

where, E represents Euclidean dimension. This equation can be used for the measurement of a fractal object in a form of either curve, area or volume. For instance, if $E = 1$, then G and δ correspond to a fractal curve. In this case, eqn (3) becomes

$$L(\delta) = L_0 \delta^{1-D} \tag{4}$$

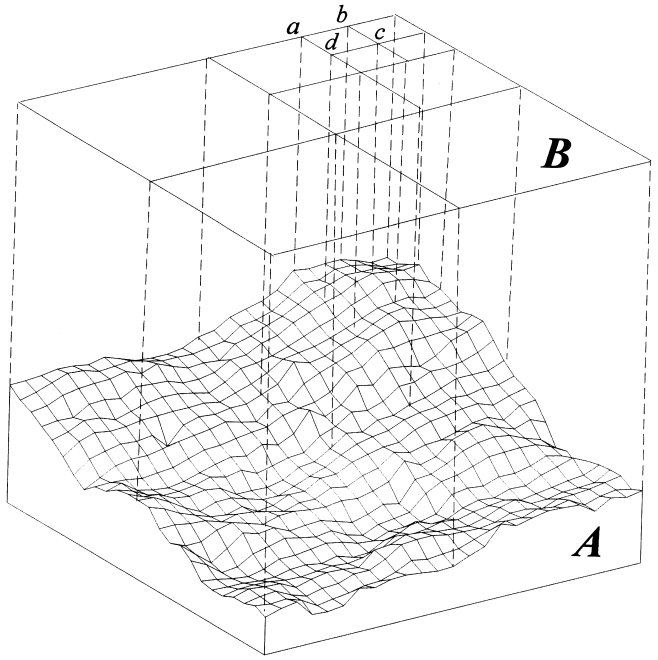
Similarly, if $E = 2$, G and δ in eqn (3) correspond to a fractal area, then eqn (3) yields

$$A_T(\delta) = A_{T0} \delta^{2-D_s} \tag{5}$$

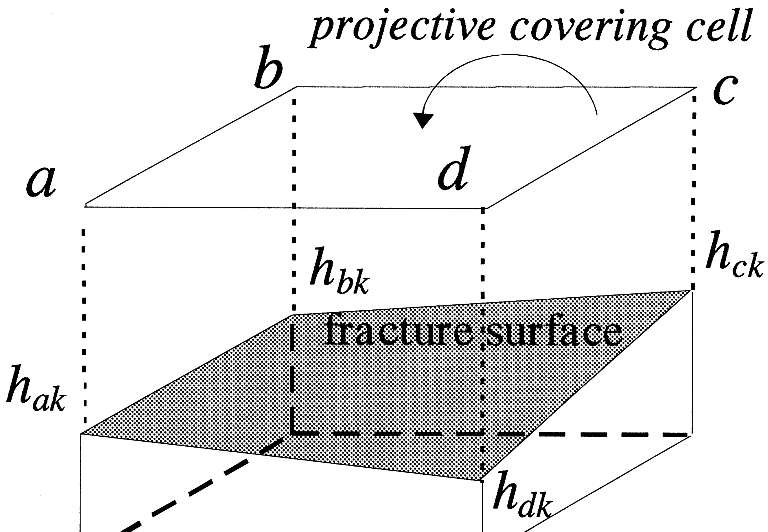
where A_{T0} denotes the apparent area of the rough surface. From eqns (2) and (5), we have the following relation

$$A_T(\delta) = \sum_{k=1}^{N(\delta)} A_k(\delta) \sim \delta^{2-D_s} \tag{6}$$

where D_s is the real fractal dimension of a rough surface. Instead of using divider of size δ to cover a fracture profile, projective covering method uses rectangle of size $\delta \times \delta$ to cover a rough surface.



(a)



(b)

Fig. 1. The projective covering method.

Although this approach is similar to divider method, it produces a real fractal dimension $D_s \in [2, 3)$ for rough surface.

3. Direct measurement of fractal dimensions of fracture surfaces

In order to make a direct measurement of fractal dimensions of fracture surfaces, the measurement technique should be taken into account in the first place. To date, the techniques developed for measuring of rough surface can be classified into mechanical and optical ones.

To avoid damages and errors to the measured surfaces caused by scratches of mechanical probe sliding along the surfaces, the laser scanner (Kwaśniewski and Wang, 1993), a non-contact optical instrument, is employed in the present study to measure fracture surfaces. Following the principle of triangular reflection, a laser beam is released from the source and forms a point on the fracture surface. The surface reflects part of the light on a position sensitive detector (PSD) in a definable angle. When the distance between the surface and the light source changes, the reflected light will be thrown to different positions on PSD. The light–electrical transfer will produce an electronic signal which is an analogue of the distance. In this way, the height of a rough surface can be measured. The measurement range in height of the scanner is 30 cm, with its accuracy of ± 7 μm, and resolution of 7.5 μm.

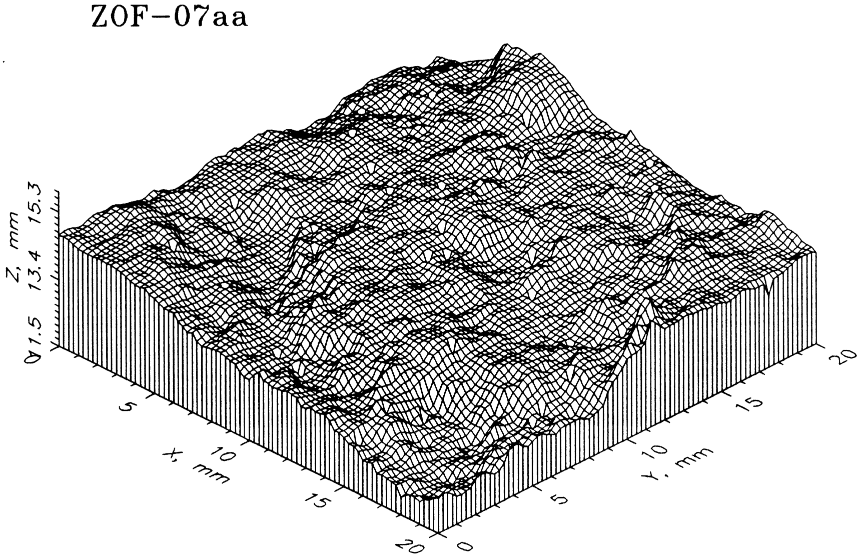
In our study, a number of shear fracture surfaces induced in sandstone by triaxial compression tests are scanned within an area of 20×20 mm^2 over 6561 points. The digital interval is 0.25 mm. As an example, Fig. 2 shows the morphology of a fracture surface by making use of the laser scanner. According to the projective covering method, the fractal dimension D_s of the fracture surface can be directly estimated from the slope β of the log–log plot $A_T(\delta)$ vs δ, i.e., $D_s = 2 - \beta$ (Fig. 3).

As shown in Fig. 3, fracture surfaces in rocks do not show strict self-similar fractal behavior. The segmental linearity of the logarithm plots in different scale sizes ($\delta = \delta_i/\delta_0$) indicates that the roughness of fracture display multi-scale fractal property. Fractal dimension of a real fracture surface in rock depends on measurement scale of the projective covering network, i.e., the smaller δ is used, the greater the fractal dimension is produced, and vice versa. The scale effect on fractal dimension of fracture surface suggests that the fracture surface may display multifractal behavior, which we will discuss in a separate paper.

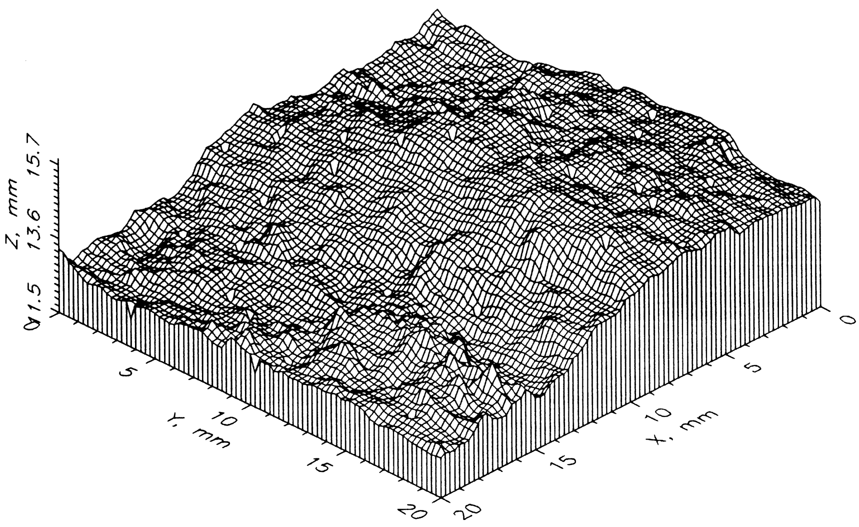
4. Comparison with fractal measurement of profiles

To verify the projective covering method, for same fracture surfaces, the fractal dimensions along individual profiles in x- and y-directions, respectively, are measured by the divider method. Fractal dimension D for a profile is calculated following eqn (4). Fractal dimensions measured in this way has the value $D \in [1,2)$ for a single profile. In order to compare fractal dimension of a fracture surface measured by PCM with that measured from profiles, let us elaborate the dimension formulae of Cartesian product of fractal sets.

Suppose E is a subset of $\mathbb{R}^n$ and F is a subset of $\mathbb{R}^m$, the Cartesian product, $E \times F$, is defined as the set of points with first coordinate in E and second coordinate in F, i.e.,

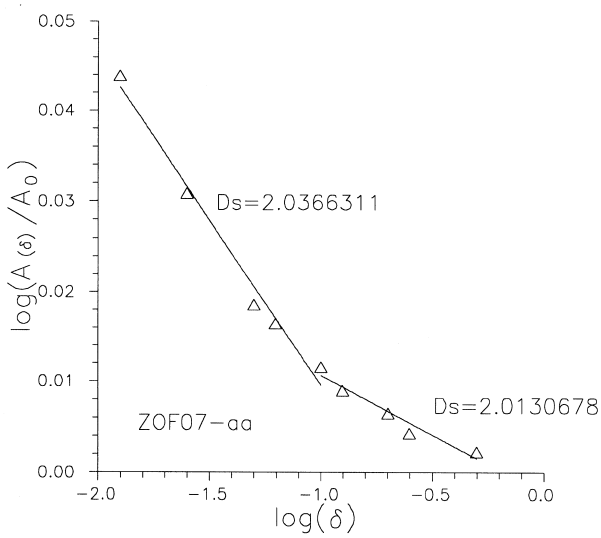


(a)

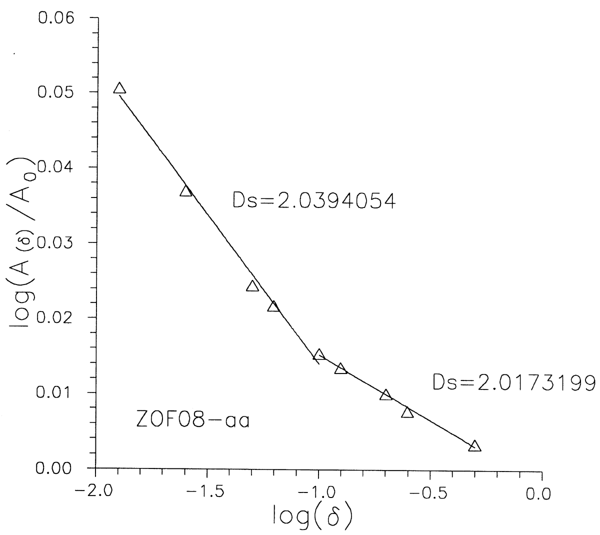


(b)

Fig. 2. Scanned fracture surfaces in rock.



(a)



(b)

Fig. 3. Estimation of fractal dimension of fracture surface.

$$E \times F = \{(x, y) \in \mathbb{R}^{n-m} : x \in E, y \in F\} \tag{7}$$

Thus if E is a unit interval in $\mathbb{R}$, and F is a unit interval in $\mathbb{R}^2$, then $E \times F$ is a unit square in $\mathbb{R}^3$ (Fig. 4). In such a case, it is obvious that (Falconer, 1990)

$$\dim(E \times F) = \dim E + \dim F \tag{8}$$

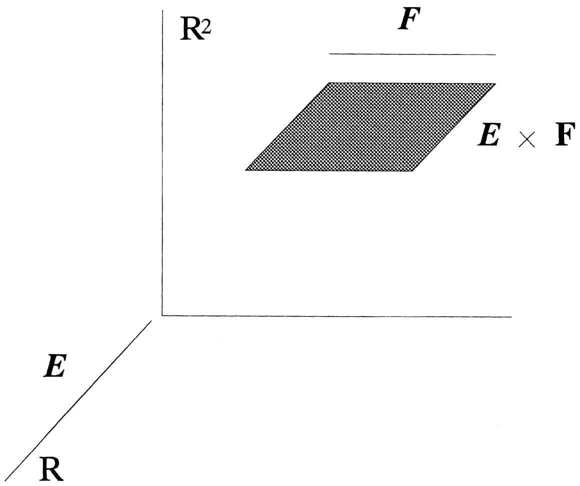


Fig. 4. The Cartesian product of a unit interval in $\mathbb{R}$ and a unit interval in $\mathbb{R}^2$ (Falconer, 1990).

using the classical definition of dimension. This holds more generally, in the 'smooth' situation, where E and F could be smooth curves, surfaces or high-dimensional manifolds. However, eqn (8) is not always valid for 'fractal' dimensions. For fractal dimensions, the most general result possible is an inequality (Falconer, 1990)

$$\dim(E \times F) \leqslant \dim E + \dim F \tag{9}$$

For simplicity, take $E \subset \mathbb{R}$ and $F \subset \mathbb{R}$. Choose number $s > \dim E$ and $t > \dim F$. Then there is a number $\delta_0 > 0$ such that E may be covered by $N_\delta(E) \leqslant \delta^{-s}$ intervals, and similarly, F may be covered by $N_\delta(F) \leqslant \delta^{-t}$ intervals of side length δ for all $\delta \leqslant \delta_0$. Thus, $E \times F$ is covered by $N_\delta(E)N_\delta(F)$ squares formed by products of these intervals with length δ, so that

$$N_\delta(E \times F) = \delta^{-\dim(E \times F)} = N_\delta(E) \times N_\delta(F) \leqslant \delta^{-s}\delta^{-t} = \delta^{-(s+t)} \tag{10}$$

recall that $s > \dim E$ and $t > \dim F$. By choosing s and t equal to dim E and dim F, the equality holds in eqn (10).

For example, a Koch fractal surface as shown in Fig. 5 is constructed by producing a Koch curve with a straight line perpendicular to the Koch curve. As is well known, the 'fractal' dimension for a straight line is **1** and fractal dimension for Koch curve is $D = 1.2619$. Following eqn (10), the product yields $D_s = 1 + 1.2169 = 2.2169$ for a Koch fractal surface.

For a real fracture surface in rock, the roughness varies in different profiles and along different directions (Wang, 1997). In order to clarify the fractal dimension under consideration, the fractal dimensions of fracture profiles within the fracture surfaces are measured by divider method along x and y directions, respectively. The fractal dimension is estimated in two orthogonal directions, respectively, by the following equations

$$L_x(\delta) = L_{x0}\delta^{1-D_x}, \quad L_y(\delta) = L_{y0}\delta^{1-D_y} \tag{11}$$

where, $L_x(\delta)$ and $L_y(\delta)$ are the real profile lengths in the x- and y-directions measured under the

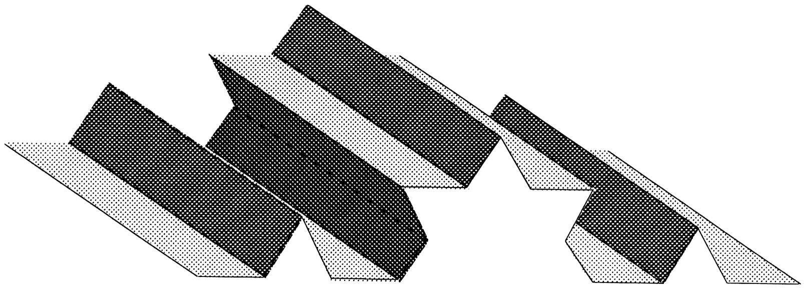


Fig. 5. Koch fractal surface formed by product of Koch curve with a straight line.

scale of δ_i by the divider method, and D_x, D_y are fractal dimensions estimated along the x- and y-directions, respectively.

To compare the results of the productive covering method with the divider method of profiles, the fractal dimension based on profiles measurements of a rough surface is then calculated by

$$D_{xy} = \frac{1}{N}\sum_{i=1}^{N}(D_{x_i}+D_{y_i}) \tag{12}$$

The results are given in Table 1, where D_x and D_y are the average values of fractal dimensions which are estimated by divider method over 81 scanning profiles in the x- and y-directions, respectively; $D_{xy}(=D_x+D_y)$ is the averaged fractal dimension of all profiles over the rough surface; and D_s in Table 1 is the fractal dimension estimated by direct projective covering method. In fact, the fractal dimension D_{xy} obtained by averaging of fractal dimensions of all profiles represents a statistical fractal property. By comparison of D_s with D_{xy}, we have $D_s \leqslant D_{xy}$. Figure 6 gives the results of D_s compared with D_{xy}. As it is shown, for smooth fracture surface D_s is slightly smaller than D_{xy}; however, the difference of D_s and D_{xy} increases with the increase of surface roughness. The general relation of the results can be expressed as follows

$$1+D_x \leqslant D_s \leqslant D_x+D_y \wedge 1+D_y \leqslant D_s \leqslant D_x+D_y \tag{13}$$

which agree very well with the theory of fractal geometry (ref. eqn (10)). The study indicates that for rougher fracture surfaces, fractal measurement based on sectional profiles may overestimate the fractal dimension of the surface. Better than the fractal measurement from sectional profile and modifying it by adding 1.0 to simulate the fracture surface as suggested by Mandelbrot (1983), the projective covering method yields more accurate and quite satisfactory result in a direct manner to estimate the fractal dimension of fracture surfaces.

5. Conclusions

In this paper, a new measurement method—the projective covering method is proposed which makes it possible to cover two-dimensional fractal objects for direct estimation of real fractal dimensions $D_s \in [2,3)$ of fracture surfaces. The results agree well with the theory of fractal geometry.

Table 1
Fractal dimensions of rock fracture surfaces

No.	Sample	D_x	D_y	D_{xy}	D_s
1	JAS01-aa	1.02620	1.018331	2.044531	2.042013
2	JAS02-aa	1.044379	1.034598	2.078977	2.0742193
3	JAS03-aa	1.043866	1.0301864	2.0740524	2.0708306
4	JAS04-aa	1.065890	1.046345	2.112235	2.100897
5	JAS05-aa	1.0509943	1.0366843	2.0876786	2.0818937
6	JAS06-aa	1.0398555	1.0269191	2.0667746	2.064701
7	JAS07-aa	1.048658	1.0313137	2.0799717	2.0734053
8	JAS08-aa	1.0492081	1.034011	2.0832191	2.0770266
9	JAS09-aa	1.0588974	1.0418709	2.1007683	2.0824751
10	JAS10-aa	1.039661	1.0265146	2.0634096	2.063206
11	JAS11-aa	1.0486502	1.0328192	2.0814694	2.0736565
12	JAS12-aa	1.0538008	1.0394035	2.0932043	2.080631
13	ZOF05-aa	1.0251804	1.0216661	2.0468465	2.0437931
14	ZOF07-aa	1.0274177	1.0186337	2.0460514	2.0421595
15	ZOF08-aa	1.0269127	1.0187078	2.0456205	2.0434385
16	STA-P41	1.0346635	1.0216785	2.056342	2.0559967
17	J1-599B	1.0581690	1.0325522	2.0907212	2.0815781
18	J2-599R	1.069016	1.0336108	2.1026268	2.0865448
19	ZOF03	1.0458004	1.0295378	2.0753382	2.0709635

D_x, D_y—average fractal dimension estimated by divider method for the profiles in x, y direction.
D_{xy}—the summation of D_x and D_y ($D_{xy} = D_x + D_y$).
D_s—fractal dimension directly estimated by projective covering methods.

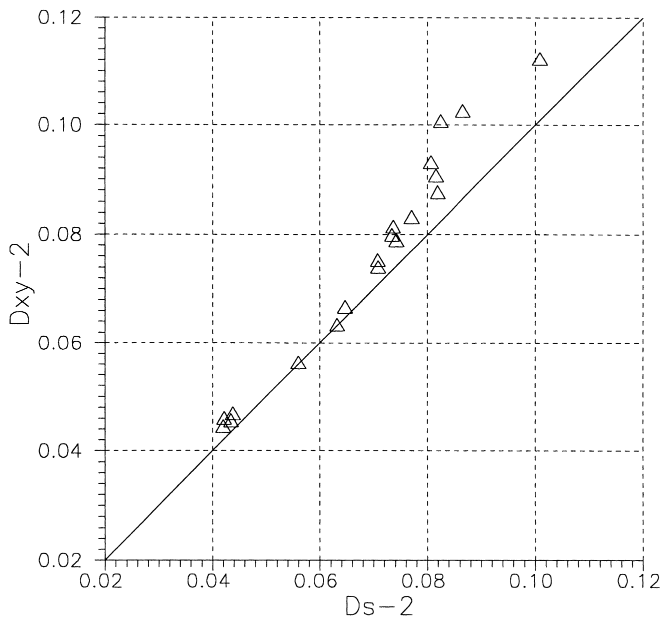


Fig. 6. Comparison of D_s with D_{xy}.

Acknowledgements

The presented work is supported by the National Distinguished Youth Science Foundation of China, the Trans-Century Programme for the Telents by the State Education Commission and China Postdoctoral Research Foundation. Dr M. A. Kwaśniewski has kindly provided help and facilities for the measurement.

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